Large c TT bar CFT

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1. Large C 极限下解 TT deformation
2. 在球对称空间中显式计算配分函数与熵
3. holography

1、Large c 极限下的求解

主要技巧：Hubbard - Stratonovich 变换、鞍点近似、Large c 近似

(1) deformation：$S[\phi] \to S[\phi] + \frac{f}{2}\int d^dx\sqrt{\gamma}\,\mathcal{O}^2$ → 一次型

则 $\mathbb{Z}^{(f)}[J] = \int D\phi\, e^{-S[\phi] - \frac{f}{2}\int d^dx\sqrt{\gamma}\,\mathcal{O}^2 + \int d^dx\sqrt{\gamma}\,J\mathcal{O}}$ J source of $\mathcal{O}$

H-S 变换：用恒等式 $1 = \sqrt{\det(-\frac{1}{f}\mathbb{1})}\int D\sigma\, e^{\frac{1}{2f}\int d^dx\sqrt{\gamma}(\sigma + f\mathcal{O})^2}$

(类似于 $1 = \sqrt{a}\int\frac{dx}{\sqrt{\pi}}e^{-a(x+b)^2}$) $1 = (\sqrt{a})^n\int\frac{d^nx}{(\sqrt{\pi})^n}e^{-a\sum_n(x_n-b)^2}$ P51

插进 $\mathbb{Z}$ 中，化 $\mathcal{O}^2$ 为 $\mathcal{O}$ 一次项

$$\mathbb{Z}^{(f)}[J] = \int D\phi\sqrt{\det(-\tfrac{1}{f}\mathbb{1})}\int d\sigma\, e^{-S[\phi] + \int d^dx\sqrt{\gamma}(J+\sigma)\mathcal{O} + \frac{1}{2f}\int d^dx\sqrt{\gamma}\,\sigma^2}$$

$$= \sqrt{\det(\ )}\int D\sigma\, e^{\frac{1}{2f}\int d^dx\sqrt{\gamma}\,\sigma^2}\underbrace{\int D\phi\, e^{-S[\phi] + \int d^dx\sqrt{\gamma}(J+\sigma)\mathcal{O}}}_{\mathbb{Z}[J+\sigma]}$$

$I = -\log\mathbb{Z}$

$e^{-I^{(0)}[J+\sigma]}$

$$= \sqrt{\ }\int d\sigma'\, e^{\frac{1}{2f}\int d^dx\sqrt{\gamma}(\sigma'-J)^2 - I^{(0)}[\sigma']}$$

$J+\sigma$ $\mathcal{O}$

鞍点近似：EoM：$J = \sigma' - \frac{f}{\sqrt{\gamma}}\frac{\delta I^{(0)}[\sigma']}{\delta\sigma'}\Big|_{\sigma'=\sigma^*} = \sigma^* + f\langle\mathcal{O}\rangle^{(0)}_{\sigma^*}$

$\langle\mathcal{O}\mathcal{O}\rangle$

$$\mathbb{Z}^{(f)}[J] = e^{-I^{(0)}[\sigma^*] + \frac{f}{2}\int d^dx\sqrt{\gamma}\langle\mathcal{O}\rangle^{(0)2}_{\sigma^*}}\sqrt{\ }\int D\xi\, e^{\frac{1}{2f}\int d^dx\sqrt{\gamma}\,\xi^2 - \frac{1}{2!}\int d^dx\,d^dy\frac{\delta^2 I^{(0)}(\sigma')}{\delta\sigma(x)\,\delta\sigma'(y)}\Big|_{\sigma^*}\xi(x)\xi(y) + \cdots}$$

$\xi = \sigma' - \sigma^*$ $\sqrt{\gamma(x)}\sqrt{\gamma(y)}\langle\mathcal{O}(x)\mathcal{O}(y)\rangle^{(0)c}_{\sigma^*}$

Large c 极限：$\langle\mathcal{O}\mathcal{O}\rangle^c \ll \langle\mathcal{O}\rangle^2$ connected

$$\mathbb{Z}^{(f)}[J] = e^{-I^{(0)}[\sigma^*] + \frac{f}{2}\int d^dx\sqrt{\gamma}\langle\mathcal{O}\rangle^{(0)2}_{\sigma^*}}$$

f $e^{f(\langle\mathcal{O}^2\rangle + \langle\mathcal{O}\mathcal{O}\rangle)}$ $e^{-I^{(0)}[\sigma^*]}$ $c\to\infty$

视 σ^* 为 $J^{(0)}$，则 $I^{(f)}[J] = I^{(0)}[J^{(0)}] - \frac{f}{2}\int d^dx\sqrt{\gamma}\langle\mathcal{O}\rangle^{(0)2}_{J^{(0)}}$

$$\begin{cases} \partial_f I^{(f)}[J^{(f)}] = \dfrac{I^{(f)}[J] - I^{(0)}[J^{(0)}]}{f} = -\dfrac{1}{2}\int d^dx\sqrt{\gamma}\langle\mathcal{O}\rangle^{(f)2}_{J^{(f)}} \ ✓ \\ \partial_f J^{(f)} = \dfrac{J^{(f)} - J^{(0)}}{f} = \langle\mathcal{O}\rangle^{(f)}_{J^{(f)}} \ ✓ \end{cases}$$

$\cancel{+I^{(f)}[J^{(f)}]}$ I

$$\partial_f I^{(f)}[J^{(0)}] = \frac{I^{(f)}[J^{(0)}] - I^{(0)}[J^{(0)}]}{f} = \frac{1}{2}\int d^dx\sqrt{\gamma}\langle\mathcal{O}\rangle^{(f)2}_{J}$$

1.2 deformation : $S[\phi,\gamma_{ij}] \to S[\phi,\gamma'_{ij}] - \frac{\delta\lambda}{2}\int d^2x\sqrt{\gamma'}\,T\bar T$ $T\bar T = T^{ij}T_{ij} - T^2 = -\epsilon^{ik}\epsilon^{jl}T_{ij}T_{kl}$

$$1 = \sqrt{\det(-\delta\lambda M_{\gamma'_{ij}})}\int Dh\, e^{\frac12\delta\lambda\int d^2x\sqrt{\gamma'}\,\epsilon^{ik}\epsilon^{jl}(h_{ij}+T_{ij})\times(h_{kl}+T_{kl})}$$

h_{ij}

$$Z'[\gamma_{ij}] = \sqrt{\quad}\int Dh\, e^{\frac12\delta\lambda\int d^2x\sqrt{\gamma'}\,\epsilon^{ik}\epsilon^{jl}h_{ij}h_{kl}}\int D\phi\, e^{-S[\phi,\gamma'_{ij}]+\delta\lambda\int d^2x\sqrt{\gamma'}\,\epsilon^{ik}\epsilon^{jl}T_{ij}h_{kl}}$$

$$\approx \quad \ldots \ldots \quad \int D\phi\, e^{-S[\phi,\gamma'^{ij}-2\delta\lambda\epsilon^{ik}\epsilon^{jl}h_{kl}]}$$

$S[\phi,\gamma'^{ij}-2\delta\lambda\epsilon^{ik}\epsilon^{jl}h_{kl}]$
$= S[\phi,\gamma'^{ij}]+\delta\lambda\int d^2x\sqrt{\gamma}\,\epsilon^{ik}\epsilon^{jl}T_{ij}h_{kl} + \delta\lambda^2\ldots$

$$= \sqrt{\quad}\int Dh\, e^{\frac12\delta\lambda\int d^2x\sqrt{\gamma}\,\epsilon^{ik}\epsilon^{jl}h_{ij}h_{kl} - I[\gamma'^{ij}-2\delta\lambda\epsilon^{ik}\epsilon^{jl}h_{kl}]}$$

$\gamma\sim J$, $T\sim O$

类似于1.1，有

$$\begin{cases} \partial_\lambda I^{[\lambda]}[\gamma_{ij}^{[\lambda]}] = \frac12\int d^2x\sqrt{\gamma^{[\lambda]}}\,T\bar T^{[\lambda]} & ① \\ \partial_\lambda \gamma^{[\lambda]ij} = -2\epsilon^{[\lambda]ik}\epsilon^{[\lambda]jl}T^{[\lambda]}_{kl} = 2(T^{[\lambda]ij} - T^{[\lambda]}\gamma^{[\lambda]ij}) & ② \end{cases}$$

$$(\partial_\lambda I^{[\lambda]})[\gamma] = -\frac12\int d^2x\sqrt{\gamma}\,T\bar T^{[\lambda]}$$

$T\bar T = T_{ij}T^{ij} - T^2$

下面求解上述方程

① 对度规变分 : $\partial_\lambda(\sqrt{\gamma^{[\lambda]}}\,T^{[\lambda]}_{ij}\,\delta\gamma^{[\lambda]ij}) = \delta(\sqrt{\gamma^{[\lambda]}}\,T\bar T^{[\lambda]})$

$\delta\sqrt{\gamma} = -\frac12\sqrt{\gamma}\,\gamma_{ij}\delta\gamma^{ij}$ ✓ $\delta\gamma_{ik} = -\gamma_{ij}\gamma_{kl}\delta\gamma^{jl}$

$\delta(T_{ij}T^{ij}) = \delta(T^{ij}T^{kl}\gamma_{ik}\gamma_{jl})$

$= (\delta T^{ij})T^{kl}\gamma_{ik}\gamma_{jl} + T^{ij}(\delta T^{kl})\gamma_{ik}\gamma_{jl} + T^{ij}T^{kl}(\delta\gamma_{ik})\gamma_{jl} + T^{ij}T^{kl}\gamma_{ik}(\delta\gamma_{jl})$

$= 2T_{ij}\delta T^{ij} - 2T_i^{\ k}T_{jk}\delta\gamma^{ij}$

$\delta T^2 = 2T\delta T = 2T_{ij}\delta(\gamma^{ij}T) - 2TT_{ij}\delta\gamma^{ij}$

$$\partial_\lambda(\sqrt{\gamma}\,T_{ij})\delta\gamma^{ij} + \sqrt{\gamma}\,T_{ij}\delta(\cancel{\partial_\lambda\gamma^{ij}}) = \sqrt{\gamma}\,[(-\tfrac12 T\bar T\gamma_{ij} - 2T_{ik}T^k_{\ j} + 2TT_{ij})\delta\gamma^{ij} + T_{ij}\delta(\cancel{2T^{ij}-2\gamma^{ij}T})]$$

由②，上式化为 $\partial_\lambda(\sqrt{\gamma}\,T_{ij}) = \sqrt{\gamma}\,(-\frac12 T\bar T\gamma_{ij} - 2T_{ik}T^k_{\ j}+2TT_{ij})$

$\partial_\lambda(\sqrt{\gamma}\,T) = -\sqrt{\gamma}\,T\bar T$

所以 $(\sqrt{\gamma}\,T)^{[\lambda]} = (\sqrt{\gamma}\,T)^{[0]} - \lambda(\sqrt{\gamma}\,T\bar T)$ const

$T = \frac{c}{24\pi}R$

又由 $\partial_\lambda(\sqrt{\gamma}\,R)=0$，及 $\sqrt{\gamma^{[0]}}\,T^{[0]} = \frac{c}{24\pi}\sqrt{\gamma^{[0]}}\,R^{[0]}$,

得 $\cancel{\sqrt{\gamma^{[\lambda]}}}\,T^{[\lambda]} + \lambda\cancel{\sqrt{\gamma^{[\lambda]}}}\,T\bar T^{[\lambda]} = \frac{c}{24\pi}\cancel{\sqrt{\gamma^{[\lambda]}}}\,R^{[\lambda]}$

即 $\boxed{T^{[\lambda]}+\lambda T\bar T^{[\lambda]} = \frac{c}{24\pi}R^{[\lambda]}}$ deformed trace relation

$\alpha_\lambda\gamma_{ij}$

2. 球对称空间

$\gamma^{[\lambda]}_{ij} = r^2\Omega_{ij}$ $R^{[\lambda]} = \frac{2}{r^2}$

最大对称空间有

$T^{[\lambda]}_{ij} = \alpha_\lambda(r)\gamma^{[\lambda]}_{ij}$, $\tilde T^{[\lambda]}_{ij} = -\alpha_\lambda\gamma^{[\lambda]}_{ij}$

代入 trace relation 中,

$$2\alpha_\lambda + \lambda(2\alpha_\lambda^2 - 4\alpha_\lambda^2) = \frac{c}{24\pi}\,\frac{2}{r^2}$$

$$\alpha_\lambda = \frac{1}{2\lambda}\left(1-\sqrt{1-\frac{\lambda c}{6\pi r^2}}\right)$$

取负根 : $\lambda=0$时 $\alpha_\lambda = \frac{c}{24\pi}\frac{1}{r^2}$

$\frac{1}{2\lambda}\left(1-\sqrt{1-\frac{\lambda c}{6\pi r^2}}\right)$

$2\lambda \quad \lambda \quad 6\pi r^2$

$$\approx \frac{1}{2\lambda}\left(1-\left(1-\frac{1}{2}\frac{\lambda c}{6\pi r^2}\right)\right)=\frac{c}{24\pi}\frac{1}{r^2}$$

度规反向flow：

$$\gamma^{[0]}_{ij} = \gamma^{[\lambda]}_{ij} - 2(-\lambda)\hat{T}^{[\lambda]}_{ij} + (-\lambda)^2 \hat{T}^{[\lambda]}_{ik}\gamma^{[\lambda]kl}\hat{T}^{[\lambda]}_{lj} = (1-\lambda\alpha_\lambda)^2\gamma^{[\lambda]}_{ij}$$

而 $\gamma^{[0]}_{ij} = r_0^2\Omega_{ij}$, $\gamma^{[\lambda]} = r^2\Omega_{ij}$

故 $r_0 = r(1-\lambda\alpha_\lambda) = \frac{1}{2}\left(1+\sqrt{1-\frac{\lambda c}{6\pi r^2}}\right)r$

CFT 生成泛函 $I^{[0]} = -\frac{c}{3}\log r_0 + C_0 = -\frac{c}{3}\log\frac{\left(1+\sqrt{1-\frac{\lambda c}{6\pi r^2}}\right)r}{2} + C_0$

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deformed: $I^{[\lambda]} = I^{[0]} + \frac{\lambda}{2}\int d^2x\sqrt{\gamma}\,T\bar{T}$

$= I^{[0]} + \frac{\lambda}{2}4\pi r^2(-2\alpha_\lambda^2)$

$= -\frac{c}{3}\log\frac{\left(1+\sqrt{1-\frac{\lambda c}{6\pi r^2}}\right)r}{2} + \frac{c}{6} - \frac{2\pi r^2}{\lambda}\left(1-\sqrt{1-\frac{\lambda c}{6\pi r^2}}\right) + C_0$

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$$\log Z^{[\mu]} = \frac{c}{3}\log\frac{\left(1+\sqrt{1+\frac{\mu c}{24\pi r^2}}\right)r}{2} - \frac{c}{6} + \frac{8\pi r^2}{\mu}\left(\sqrt{1+\frac{\mu c}{24\pi r^2}}-1\right) - C_0 \quad \checkmark$$

$$\log Z^{[\mu]} = \frac{c}{3}\log\left(\sqrt{\frac{24\pi}{\mu c}}\,r + \sqrt{1+\frac{24\pi r^2}{\mu c}}\right) + \frac{8\pi r^2}{\mu}\left(\sqrt{1+\frac{\mu c}{24\pi r^2}}-1\right) \quad \checkmark$$

$$r\partial_r \log Z = \frac{16\pi r^2}{\mu}\left(\sqrt{1+\frac{\mu c}{24\pi r^2}}-1\right) = \int d^2x\sqrt{\gamma}\,T$$

(3.11)

$+ f(\mu)$

$\mu \to 0$.

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1. 二阶修正

2. $\frac{1}{c}$ 的修正